Classification of Tripartite Entangled States With Machine Learning

Naema Asif, Minseok Choi and Hyundong Shin
Department of Electronics and Information Convergence Engineering, Kyung Hee University, Korea
Email: hshin@khu.ac.kr

***Abstract*—Genuine Multipartite Entangled (QME); Greenberger–Horne–Zeilinger state and W state play significant roles in the quantum communication and cryptography protocols. However, due to the complex nature, their detection comes forth as a challenge. For this purpose, we extract features from Mermin and Svetlichny inequalities. We then train an artificial neural network (ANN) to obtain a classifier model. The method is faster and efficient than quantum state tomography (QST) as it uses partial information rather than the entire density matrices.**

I. Introduction

Greenberger–Horne–Zeilinger (GHZ) state and W states are important in the field of quantum computing and quantum information because they represent different types of entanglement that can be exploited for various applications. The classification of these states is important because it allows researchers to understand the different types of entanglement that can occur in a quantum system and to determine which type of entanglement is most suitable for a particular application [1], [2].

For example, GHZ states are known for their high level of entanglement, which makes them useful for certain types of quantum communication and computation tasks [3]. In particular, GHZ states have been used in the development of quantum error correction codes, which are used to protect quantum information from errors that can occur during quantum computations [4]. W states, on the other hand, are known for their weaker entanglement, which makes them less suitable for certain types of quantum communication and computation tasks. However, W states have other potential applications, such as in the development of quantum networks and in the implementation of quantum algorithms [5].

Overall, the classification of GHZ and W states is important because it helps researchers to understand the different types of entanglement that can occur in a quantum system and to determine which type of entanglement is most suitable for a particular application.

An artificial neural network (ANN) is a type of machine learning algorithm that is inspired by the structure and function of the human brain. ANNs are composed of interconnected ”neurons” that can process and transmit information. They are commonly used for a variety of tasks, including classification.

In this work, we use an artificial neural network along with partial information extracted from a quantum dataset based on the Bell-type inequalities for tripartite quantum states as features. We train the model with this data to develop a classifier capable of detecting the GHZ and W class of tripartite quantum states [6]–[8].

II. Result

In this section, we explore the use of an artificial neural network for the classification of GME tripartite quantum states. To begin, we generate a quantum dataset having $200,000$ GHZ class pure states and $200,000$ W class pure states. The maximally entangled GHZ state and GHZ class pure quantum states are denoted as

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \tag{1}$$

$$|\psi_{GHZpure}\rangle = \cos\theta\,|000\rangle + \sin\theta\,|111\rangle\,, \tag{2}$$

while the maximally entangled W state and W class pure states are expressed as

$$|\psi_{W}\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle), \tag{3}$$

$$\begin{aligned}|\psi_{Wpure}\rangle = &\cos\theta\cos\phi\,|001\rangle + \cos\theta\sin\phi\,|010\rangle \\ &+ \sin\theta\,|100\rangle\,,\end{aligned} \tag{4}$$

where $\theta \in (0, \pi)$ and $\phi \in (0, 2\pi)$. The label 0 denotes GHZ class while 1 denotes W class.

In order to obtain features from the dataset, we extract the terms of the Mermin and Svetlichny inequalities which are the multipartite counterpart of Bell's inequalities respectively,

$$\langle \mathbf{a}_0\mathbf{b}_0\mathbf{c}_0\rangle - \langle \mathbf{a}_0\mathbf{b}_1\mathbf{c}_1\rangle - \langle \mathbf{a}_1\mathbf{b}_0\mathbf{c}_1\rangle - \langle \mathbf{a}_1\mathbf{b}_1\mathbf{c}_0\rangle \leq 2, \tag{5}$$

$$\begin{aligned}&-\langle \mathbf{a}_0\mathbf{b}_0\mathbf{c}_0\rangle + \langle \mathbf{a}_0\mathbf{b}_0\mathbf{c}_1\rangle + \langle \mathbf{a}_0\mathbf{b}_1\mathbf{c}_0\rangle + \langle \mathbf{a}_0\mathbf{b}_1\mathbf{c}_1\rangle \\ &+ \langle \mathbf{a}_1\mathbf{b}_0\mathbf{c}_0\rangle + \langle \mathbf{a}_1\mathbf{b}_0\mathbf{c}_1\rangle + \langle \mathbf{a}_1\mathbf{b}_1\mathbf{c}_0\rangle - \langle \mathbf{a}_1\mathbf{b}_1\mathbf{c}_1\rangle \leq 4.\end{aligned} \tag{6}$$

where $\langle . \rangle$ denotes the expectation value of the operators. These inequalities are generally violated by the entangled states, however, such cases exist where the inequality is not violated even though the state is entangled. We consider the following measurement settings based on Pauli operators for the three qubits:

$$\begin{aligned}&\mathbf{a}_0 = \sigma_x, \qquad &&\mathbf{b}_0 = -\sigma_y, \qquad &&\mathbf{c}_0 = \sigma_x, \\ &\mathbf{a}_1 = \sigma_z, \qquad &&\mathbf{b}_1 = \sigma_z, \qquad &&\mathbf{c}_1 = \sigma_z.\end{aligned}$$

Here σ_x, σ_y and σ_z corresponds to Pauli X, Pauli Y and Pauli Z, respectively. Our feature vector results in the following:

$$\begin{aligned}&\{\langle \mathbf{a}_0\mathbf{b}_0\mathbf{c}_0\rangle, \langle \mathbf{a}_0\mathbf{b}_0\mathbf{c}_1\rangle, \langle \mathbf{a}_0\mathbf{b}_1\mathbf{c}_0\rangle, \langle \mathbf{a}_0\mathbf{b}_1\mathbf{c}_1\rangle, \\ &\langle \mathbf{a}_1\mathbf{b}_0\mathbf{c}_0\rangle, \langle \mathbf{a}_1\mathbf{b}_0\mathbf{c}_1\rangle, \langle \mathbf{a}_1\mathbf{b}_1\mathbf{c}_0\rangle, \langle \mathbf{a}_1\mathbf{b}_1\mathbf{c}_1\rangle\ \}\end{aligned} \tag{7}$$

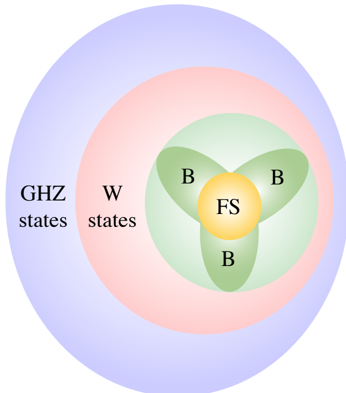

Fig. 1. Tripartite Quantum states depicting (a) GME states: GHZ and W states, (b) biseparable states, and (c) fully separable states.

The quantum dataset is divided into 80% training set and 20% testing set for the neural network training. The training set in further split into 80% train and 20% validation data. Both the Mermin and the Svetlichny features of the training data along with their labels are input separately into a neural networks having input layer with 4 and 8 nodes, a hidden layer and an output layer with a single node, respectively. The number of hidden neurons is set as $\{0, 5, 10, 15, 20\}$ neurons to observe the performance of multiple classifiers. All the models use binary cross entropy as its loss function along with RMSProp as the optimizer. The activation function for the hidden layer is set as ReLu while sigmoid is set for the output layer as we are performing binary classification. All the models are trained and then tested for the performance by the test set.

We observe that for a simple model having no hidden neurons, we obtain a 78.76% accuracy for the Mermin features and 93.08% accuracy for the Svenlichny features. In other words, four features are not sufficient to classify states using a neural network with no hidden layer. With the introduction of the number of hidden neurons, we observe a drastic improvement in the performance of the classifiers as shown in table I and table II. A classifier with 10 hidden neurons gives us an accuracy of 99.93% trained with Mermin features while and accuracy of 99.96% trained with Svetlichny features. Therefore, we observe that classification of GHZ and W states can be performed with only four features with the help of neural network having a hidden layer.

TABLE I
CONFUSION MATRICES FOR PREDICTION WITH MERMIN FEATURES

No. of hidden neurons	0	5	10	15	20
Truly GHZ States	28954	39585	39985	40005	40040
Falsely GHZ States	5901	0	0	20	105
Truly W States	34059	39960	39960	39940	39885
Falsely W States	11086	455	55	35	0
Accuracy	78.76	99.43	99.93	99.93	99.86

TABLE II
CONFUSION MATRICES FOR PREDICTION WITH SVETLICHNY FEATURES

No. of hidden neurons	0	5	10	15	20
Truly GHZ States	34489	39983	39986	39952	39979
Falsely GHZ States	0	0	0	0	0
Truly W States	39980	39980	39980	39980	39980
Falsely W States	5531	37	34	68	41
Accuracy	93.08	99.95	99.96	99.92	99.95

III. CONCLUSION

We studied the use of variations of the Bell-type inequality (specifically, the Mermin and Svetlichny inequalities) as features for classifying GHZ and W quantum states using artificial neural networks (ANNs). We found that models with hidden layers performed better than simple models without hidden layers and that the performance of the Mermin and Svetlichny features was similar when using models with hidden layers. We also found that classical machine learning techniques can be helpful in quantum state classification problems as long as a suitable training dataset is available. Overall, our results suggest that the Bell-type inequalities can be effective features for classifying genuine multipartite entangled states like GHZ and W states using ANNs.

ACKNOWLEDGMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No.2019R1A2C2007037 and 2022R1A4A3033401) and by the MSIT (Ministry of Science and ICT), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2021-0-02046) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation).

REFERENCES

[1] W. Dür, G. Vidal, and J. I. Cirac, "Three qubits can be entangled in two inequivalent ways," *Phys. Rev. A*, vol. 62, p. 062314, Nov. 2000.
[2] D. Cruz, R. Fournier, F. Gremion, A. Jeannerot, K. Komagata, T. Tosic, J. Thiesbrummel, C. L. Chan, N. Macris, M.-A. Dupertuis, and C. Javerzac-Galy, "Efficient Quantum Algorithms for GHZ and W States, and Implementation on the IBM Quantum Computer," *Adv. Quantum Technol.*, vol. 2, no. 5-6, p. 1900015, 2019.
[3] S. Ghosh, G. Kar, A. Roy, D. Sarkar, and U. Sen, "Entanglement teleportation through GHZ-class states," *New J. of Phys.*, vol. 4, no. 1, p. 48, Jul. 2002.
[4] M. Moreno, A. Fonseca, and M. Cunha, "Using three-partite GHZ states for partial quantum error detection in entanglement-based protocols," *Quantum Inf. Process.*, vol. 17, p. 191, Jun. 2018.
[5] S.-B. Zheng, "Splitting quantum information via W states," *Phys. Rev. A*, vol. 74, p. 054303, Nov. 2006.
[6] Y.-C. Ma and M.-H. Yung, "Transforming bell's inequalities into state classifiers with machine learning," *npj Quantum Inf.*, vol. 4, May 2017.
[7] J. Gao, L.-F. Qiao, Z.-Q. Jiao, Y.-C. Ma, C.-Q. Hu, R.-J. Ren, A.-L. Yang, H. Tang, M.-H. Yung, and X.-M. Jin, "Experimental machine learning of quantum states," *Phys. Rev. Lett.*, vol. 120, p. 240501, Jun. 2018.
[8] S. M. A. Rizvi, N. Asif, M. S. Ulum, T. Q. Duong, and H. Shin, "Multiclass classification of metrologically resourceful tripartite quantum states with deep neural networks," *Sensors*, vol. 22, no. 18, 2022.